２０２４年度機械工学専攻

大学院修士課程入学試験問題

「機械工学」（第１部）

試験日時：２０２３年8月２９日（火）　９：００～１１：００

注意事項

1．試験開始の合図があるまで，この問題冊子を開かないこと．
2．問題は問題１から問題２までである．全問に解答すること．
3．問題の落丁，乱丁，あるいは印刷不鮮明な箇所があれば申し出ること．
4．答案用紙は４枚配付される．枚数を確認し，過不足があれば申し出ること．
5．問題ごとに２枚の答案用紙を用いて解答すること．設問Ⅰ，Ⅱに分かれている問題は，設問ごとに１枚の答案用紙を用いて解答すること．設問Ⅲまである場合は，問題冒頭の指示に従うこと．解答を表面で書ききれない時は，裏面にわたってもよい．なお，それでも解答するスペースが不足する場合は答案用紙を与えるので申し出ること．
6．答案用紙の指定された箇所に，自分の受験番号，その答案用紙で解答する問題番号を記入すること．記入もれの場合は採点されないことがある．なお，科目名欄には「機械工学（第１部）」と記入すること．答案用紙の右端にある「　／of 」については，答案用紙を追加しない場合は空欄のままでよい．但し答案用紙を追加した場合は，問題ごとの枚数を記載する．
7．解答に関係のない記号や符号を記入した答案は無効となることがある．
8．答案用紙は，解答ができなかった分も含め，全てを提出すること．
9．下書き用紙は２枚配付される．左上に自分の受験番号を記入すること．
10．下書き用紙は，使用しなかった分も含め，２枚全部を提出すること．
11．問題冊子は持ち帰ってよい．

（白紙）

問題　1

下記の I, II の両方について解答せよ．なお，I の解答に答案用紙 1 枚を，II の解答に答案用紙 1 枚を，それぞれ用いること．

I.　図 1-1 に示す$p-v$ 線図で表される熱機関サイクルを考える．作動流体は定圧比熱c_p ，比熱比κ の理想気体とする．ただし，c_p とκ は定数である．この熱機関サイクルは，圧力p_1 とp_0（$p_1 > p_0$）における二つの定圧過程（状態a → 状態b，状態c → 状態d)，および二つの可逆断熱過程（状態b → 状態c，状態d → 状態a）により構成される．また，状態a，b，c，d における温度をそれぞれT_a，T_b，T_c，T_d とする．

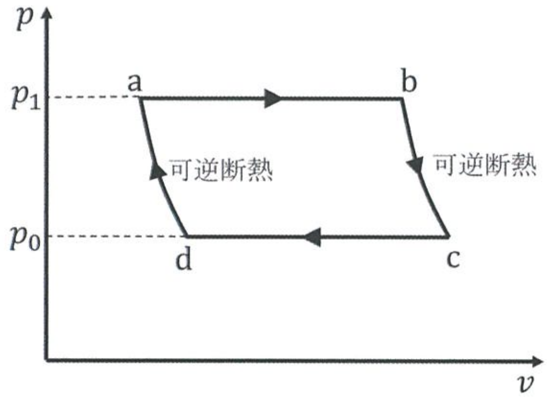

図 1-1

以下の設問に答えよ．

(1)　この熱機関サイクルの効率η をT_a，T_b，T_c，T_d を用いて表せ．

(2)　設問(1)で表した効率η をp_0，p_1，κ を用いて表せ．ただし，導出過程も示すこと．

(3)　この熱機関サイクルにおいて，最高温度および最低温度となる状態を状態a，b，c，d の中から選べ．また，この熱機関サイクルの$T-s$ 線図を作成せよ．

(4)　この熱機関サイクルと同じ最高温度と最低温度の間で動作するカルノーサイクルの効率η_c が，この熱機関サイクルの効率η よりも高いことを，式または$T-s$ 線図を利用して説明せよ．

II. 比熱c，密度ρ，および熱伝導率λ の均質媒体中の一次元非定常熱伝導を考える．ただし，c，ρ，λ は定数とする．媒体の長さはL，媒体の長さ方向と垂直な断面は単位面積である．熱は媒体の長さ方向に伝わるものとする．熱が伝わる方向にx 軸をとり，x 軸の原点は媒体の片方の端にあるとする．媒体は両端，$x = 0$，および$x = L$ においてのみ，外部と熱のやり取りをする．時刻t，位置x における温度を$T(t, x)$ とする．以下の設問に答えよ．

(1) 微小時間δt の間に，媒体のx と$x + \delta x$ の間の微小部分が熱伝導により受け取る熱量，および，微小時間δt の間に，媒体のx と$x + \delta x$ の間の微小部分を温度δT だけ上昇させるのに必要な熱量をc，ρ，λ，δx，δt，およびT の一次偏導関数，二次偏導関数の中から必要なものを用いて表せ．

(2) 設問(1)で求めた二つの熱量が等しくなることから成り立つ偏微分方程式を求めよ．

(3) 媒体（$0 \leq x \leq L$）の熱伝導による温度変化を考える．以下の条件を満たす設問(2)で求めた偏微分方程式の解$T(t, x)$ を求めよ．

初期条件（図 1-2）: $T(0, x) = (T_{\mathrm{H}} - T_{\mathrm{L}}) \sin(\pi x / L) + T_{\mathrm{L}}$

境界条件 : $T(t, 0) = T(t, L) = T_{\mathrm{L}}$

解はT_{H}，T_{L}，L，τ を用いて表せ．ただし，T_{H} とT_{L} は定数（$T_{\mathrm{H}} > T_{\mathrm{L}} > 0$），$\tau$ は特性時間であり，$\tau = \rho c L^2 / \lambda$ とする．

(4) 設問(3)と同じ条件において，媒体の単位時間あたりのエントロピー変化のうち放熱に起因する量を時間の関数として示せ．また，このとき，媒体の単位時間あたりのエントロピー変化量，そのうちの放熱に起因する量，および 0（ゼロ）の三つの値の大小関係を示せ．

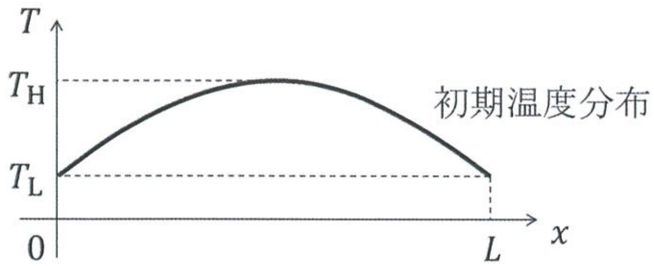

図 1-2

問題　2

下記の I, II の両方について解答せよ．なお，I の解答に答案用紙 1 枚を，II の解答に答案用紙 1 枚を，それぞれ用いること．

I．速度 U_0 で一様な空気の流れの下流に置かれた風車のモデルとして，図 2-1 に示す単純化された系を考える．空気の密度 ρ は一定とする．風車は，流れ方向（x 軸方向）の厚さを無視して，面積 A_{T} の断面 T に置き換えて考えることにする．断面 T を通過する空気の流速は U_{T} である．図 2-1 の破線は，断面 T を通過する流れの流管を示している．断面 0 を風車よりも十分上流の位置に，断面 1 を風車よりも十分下流の位置にとる．断面 0 および断面 1 における圧力は大気圧とみなせる．それぞれの断面における流管の面積は A_0, A_1 である．断面 0 および断面 1 における流速はそれぞれ U_0，U_1 である．流れは定常であり，流管内の任意の断面において流速は均一である．また，流れの損失は無視できる．風車の効率 η は，風車が流体から受ける仕事率を $\dot{W}$ として，

$$\eta = \frac{\dot{W}}{\frac{1}{2}\rho A_{\mathrm{T}} {U_0}^3}$$

と定義される．以下の設問に答えよ．

(1) 断面 0 から断面 1 までの流管を検査体積にとり，x 軸方向の運動量保存則を考えることで，風車が流体から受ける力 F を，ρ，A_0，U_0，U_1 を用いて表せ．ただし，流管の表面における圧力は大気圧とみなしてよい．

(2) エネルギー保存則より，$\dot{W}$ を ρ，A_{T}，U_0，U_1，U_{T} を用いて表せ．

(3) $FU_{\mathrm{T}} = \dot{W}$ の関係を利用して，U_{T} を U_0，U_1 を用いて表せ．

(4) η を U_0，U_1 を用いて表せ．

(5) η の最大値を求めよ．

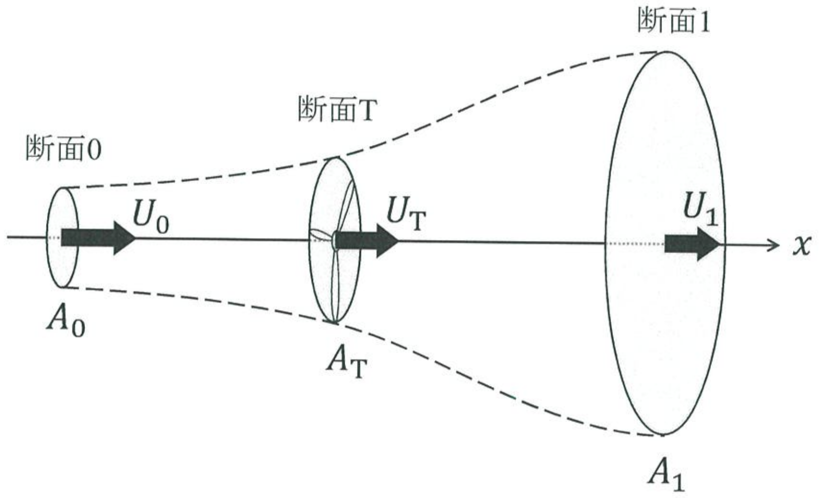

図 2-1

II. 図 2-2 に示すような水平面に対して角度θを持つ傾斜壁面上を流れる二次元液膜流れを考える．液膜を構成する液体は非圧縮性であり，密度ρと粘度μは一定である．流れは定常層流であり十分に発達している．液膜の厚さδは一定である．重力は鉛直下向きに作用し，重力加速度の大きさをgとする．座標系$(x,\ y)$の原点を傾斜壁面上にとり，x軸を傾斜壁面に接する流れ方向に，y軸を壁面垂直方向にとる．各方向に対応する速度成分をu, vとする．液膜表面における圧力は周囲の大気圧p_0と等しい．また，液膜表面に働くせん断応力は無視する．以下の設問に答えよ．

(1) 液膜内部におけるvの空間分布を求めよ．導出過程も示せ．

(2) 液膜内部における圧力pの空間分布を求めよ．導出過程も示せ．

(3) 図 2-2 の検査体積 ABCD におけるx方向の運動量保存則に基づき，傾斜壁面に働くせん断応力τ_wをg, δ, θ, ρを用いて表せ．導出過程も示せ．

(4) 液膜内部の平均流速U_0を用いて，無次元の摩擦係数c_fは次式で定義される．

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho {U_0}^n}$$

適切な整数nの値を求めよ．

(5) 設問(4)で定義したc_fをg, U_0, δ, θを用いて表せ．

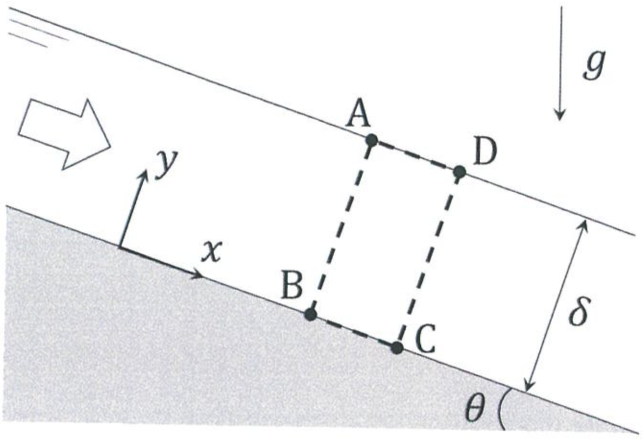

図 2-2

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FY2024 Department of Mechanical Engineering

Master Course Program Entrance Examination

“Mechanical Engineering” (Part 1)

２０２３/８/２９(Tuesday)　９：００～１１：００

Instructions

1. Do not open the exam booklet until you are instructed to begin.
2. Answer all Questions in Problems 1 and 2.
3. If you find some incomplete printing or collating, report them to the proctor.
4. Make sure that you have all 4 answer sheets. Let the proctor know otherwise.
5. Use 2 answer sheets for each Problem. If there are Questions I and II in a Problem, use one answer sheet for one Question. If there are Questions I , II and III in a Problem, follow the instruction at the top of the Problem. If the space on the front side of the answer sheet is not enough, you may also use the backside. If the space is still not enough, ask the proctor for an additional answer sheet.
6. On each answer sheet, write your examinee number (candidate number) and the Problem number in the designated boxes. If you fail to do so, the answer sheet may not be graded. Write “Mechanical Engineering (Part 1)” in “Subject”. Leave “(／ of)” blank unless you use an additional answer sheet for the Problem.
7. Answer sheets with symbols or signs that are not related to the answers may be judged invalid.
8. Hand in all the answer sheets even if you have not used them.
9. You are provided with 2 worksheets. Write your examinee number (candidate number) on the upper left corner of each worksheet.
10. Hand in both worksheets even if you have not used them.
11. You may take home the exam booklet.

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Problem 1

Answer both of the following Questions I and II. Use one answer sheet for Question I and use another answer sheet for Question II.

I. Consider a heat engine cycle represented by the $p - v$ diagram, as shown in Figure 1-1. The working fluid is an ideal gas with a specific heat at constant pressure c_p and specific heat ratio κ. Here, c_p and κ are constant. The heat engine cycle consists of two constant pressure processes (State a $\rightarrow$ State b and State c $\rightarrow$ State d) at pressures p_1 and p_0 ($p_1 > p_0$), and two reversible adiabatic processes (State b $\rightarrow$ State c and State d $\rightarrow$ State a). The temperatures at States a, b, c, and d are T_a, T_b, T_c, and T_d, respectively.

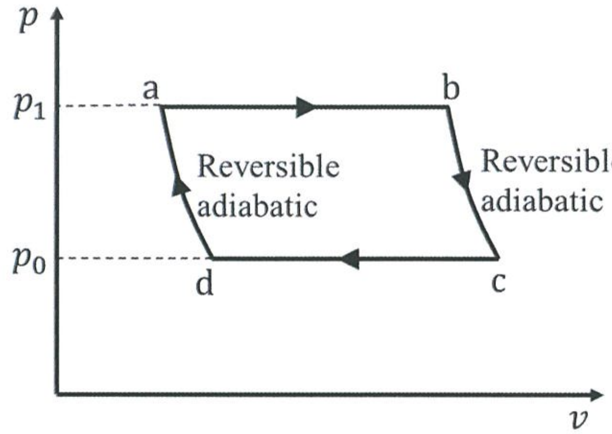

Figure 1-1

Answer the following questions.

(1) Express the efficiency of the heat engine cycle η using T_a, T_b, T_c, and T_d.

(2) Express the efficiency η obtained in Question (1) using p_0, p_1, and κ. Show also the steps of the derivation process leading to your answer.

(3) Choose the states among States a, b, c, and d having the maximum and minimum temperatures in the heat engine cycle. Moreover, also draw a $T-s$ diagram for this heat engine cycle.

(4) Using equations or a $T-s$ diagram(s), show that the efficiency of the Carnot cycle η_c operating between the same maximum and minimum temperatures of this heat engine cycle is higher than the efficiency of this heat engine cycle η.

II. Consider one-dimensional unsteady heat conduction in a homogeneous medium with specific heat c, density ρ, and thermal conductivity λ, where c, ρ, and λ are constant. The length of the medium is L, and the cross section perpendicular to the direction of the medium length has a unit area. Assume that heat is conducted in the direction of the medium length. Let the x-axis be aligned to the direction of heat conduction with its origin set at one end of the medium. The medium exchanges heat with the external environment only through its both ends, $x = 0$ and $x = L$. Let $T(t, x)$ be the temperature at time t and position x. Answer the following questions.

(1) Express the amount of heat transferred by heat conduction into an infinitesimal volume between x and $x + \delta x$ of the medium during an infinitesimal time δt and the amount of heat required to increase the temperature of this infinitesimal volume between x and $x + \delta x$ of the medium by δT in an infinitesimal time δt, using the necessary parameters among c, ρ, λ, δx, δt, and the first and second partial derivatives of T.

(2) Find the partial differential equation that satisfies the conditions for the two amounts of heat obtained in Question (1) to be equal.

(3) Consider the temperature change in time due to heat conduction of the medium ($0 \leq x \leq L$). Find the solution $T(t, x)$ of the partial differential equation found in Question (2) that satisfies the following conditions.

Initial condition (Figure 1-2): $T(0, x) = (T_{\mathrm{H}} - T_{\mathrm{L}}) \sin(\pi x / L) + T_{\mathrm{L}}$

Boundary conditions: $T(t, 0) = T(t, L) = T_{\mathrm{L}}$

Express the solution using T_{H}, T_{L}, L, and τ. Here, T_{H} and T_{L} are constant ($T_{\mathrm{H}} > T_{\mathrm{L}} > 0$), and τ is the characteristic time, defined as $\tau = \rho c L^2 / \lambda$.

(4) Under the same conditions as Question (3), express the portion of the entropy change per unit of time of the medium attributed to heat release as a function of time. In addition, order the three different

values calculated under the same conditions from the least to the greatest: the entropy change per unit of time of the medium, the portion of this entropy change per unit of time that is attributed to heat release, and zero.

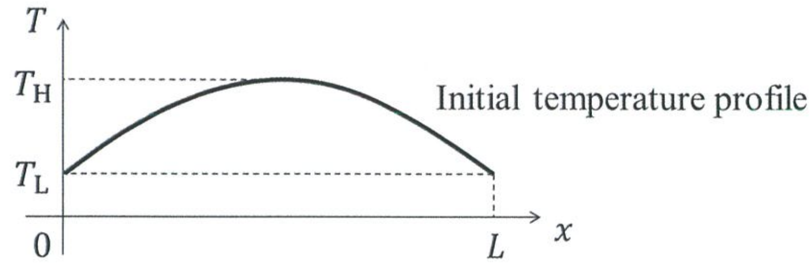

Figure 1-2

Problem 2

Answer both of the following Questions I and II. Use one answer sheet for Question I and use another answer sheet for Question II.

I. The simplified system shown in Figure 2-1 is used to model a wind turbine placed in the downstream of a uniform air flow with a velocity U_0. The density ρ of the air is constant. By neglecting the thickness of the wind turbine in the flow direction (the x-direction), the wind turbine is replaced with Cross-section T with an area A_T. The air flows through Cross-section T with a velocity U_T. The dashed lines in Figure 2-1 illustrate the stream tube of the air flow passing through Cross-section T. Cross-sections 0 and 1 are located at the far upstream and downstream from the wind turbine, respectively. The pressures at Cross-sections 0 and 1 are considered to be the atmospheric pressure. The areas of the stream tube at Cross-sections 0 and 1 are A_0 and A_1, respectively. The flow velocities at Cross-sections 0 and 1 are U_0 and U_1, respectively. The air flow is steady and has a uniform velocity distribution at any cross-section in the stream tube. The energy loss of the flow is negligible. The efficiency of the wind turbine η is given by

$$\eta = \frac{\dot{W}}{\frac{1}{2}\rho A_\mathrm{T} {U_0}^3},$$

where $\dot{W}$ is the rate of work done by the flow on the wind turbine. Answer the following questions.

(1) By applying the law of conservation of momentum in the x-direction to the control volume defined by the stream tube between Cross-sections 0 and 1, express the force F acting upon the wind turbine by the air flow using ρ, A_0, U_0, and U_1. The pressure on the surface of the stream tube is considered to be the atmospheric pressure.

(2) By applying the energy conservation law, express $\dot{W}$ using ρ, A_T, U_0, U_1, and U_T.

(3) Using the relationship $FU_{\mathrm{T}} = \dot{W}$, express U_{T} in terms of U_0 and U_1.

(4) Express η using U_0 and U_1.

(5) Find the maximum value of η.

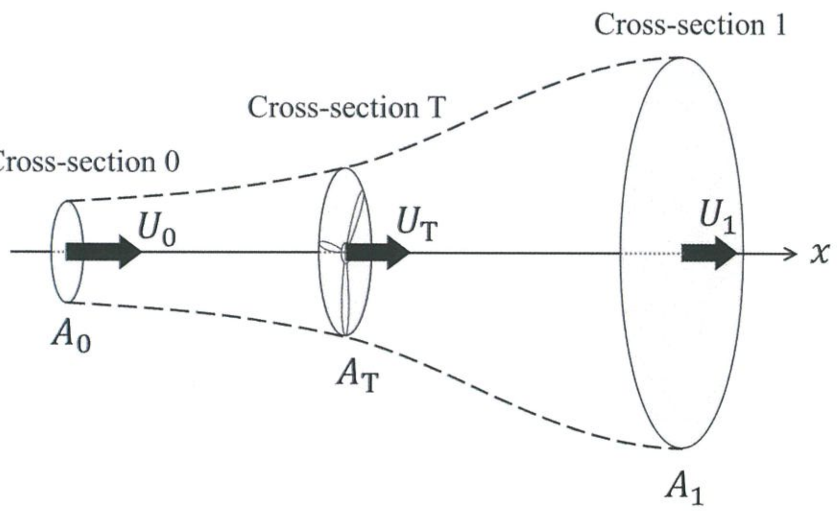

Figure 2-1

II. Consider a two-dimensional liquid film flowing along an incline making an angle θ with respect to the horizontal as shown in Figure 2-2. The fluid in the liquid film is incompressible, and the density ρ and the coefficient of viscosity μ are constant. The flow is a steady laminar flow and is fully developed. The liquid film thickness δ is constant. The gravity acts in the vertically downward direction, and the magnitude of the gravitational acceleration is g. The origin of the coordinate system (x, y) is on the incline. The x-axis is in the streamwise direction tangential to the incline and the y-axis is along the wall-normal direction. The corresponding velocity components are denoted by u and v, respectively. The pressure at the liquid film surface is the same as the atmospheric pressure p_0. The shear stress acting on the liquid film surface is negligible. Answer the following questions.

(1) Obtain the spatial distribution of v inside the liquid film. Describe the steps of the derivation process of your answer.

(2) Obtain the spatial distribution of the pressure p inside the liquid film. Describe the steps of the derivation process of your answer.

(3) By applying the momentum conservation law in the x-direction to the control volume ABCD as defined in Figure 2-2, express the shear stress τ_w acting on the incline using g, δ, θ, and ρ. Describe the steps of the derivation process of your answer.

(4) Using the averaged velocity U_0 inside the liquid film, the dimensionless friction coefficient c_f is defined as follows:

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho {U_0}^n}.$$

Find the value of an appropriate integer n.

(5) Express c_f defined in Question (4) using g, U_0, δ, and θ.

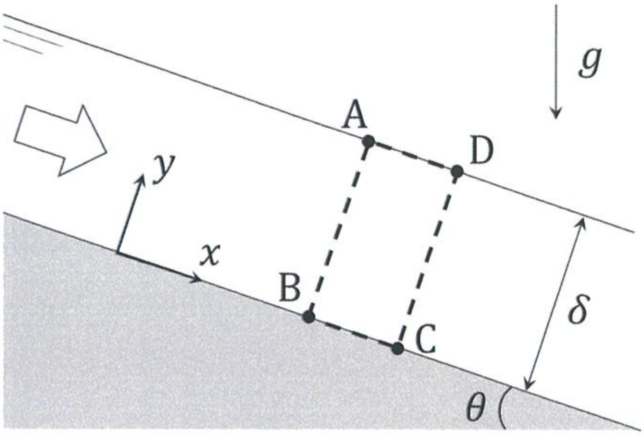

Figure 2-2

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